Photo Stitching

Panoramas from Multiple Images

Computer Vision

CS 543 / ECE 549

University of Illinois

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So far, we've looked at what can be done with one image

- Recover basic geometry using vanishing points
- Find image boundaries and segment objects
- Categorize images
- Find specific objects and detect objects that are part of some category

What can we get from multiple images?

What can we get from multiple images?

- Bigger, Better, Brighter, Sharper images
  - Panoramas
  - Increased dynamic range
  - Super-resolution
  - Reduced noise/blur

Product example: http://www.vreveal.com/

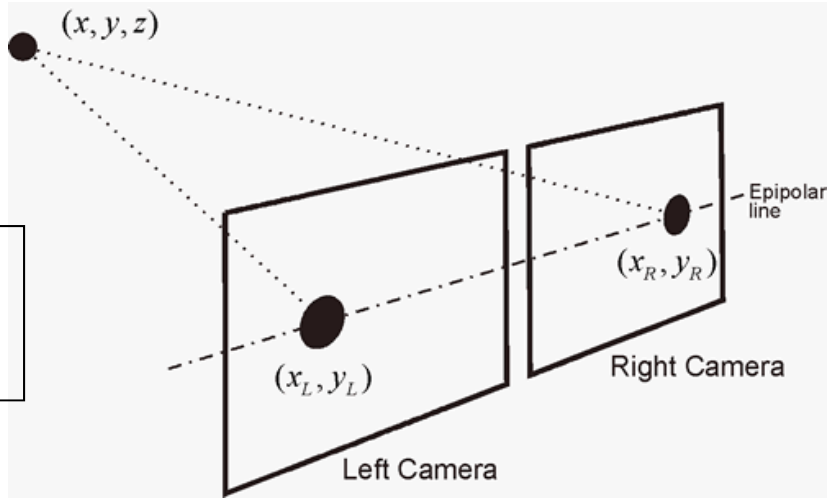
What can we get from multiple images?

- Bigger, Better, Brighter, Sharper images
  - **Panoramas ⟵ today**
  - Increased dynamic range
  - Super-resolution
  - Reduced noise/blur

Product example: http://www.vreveal.com/

What can we get from multiple images?

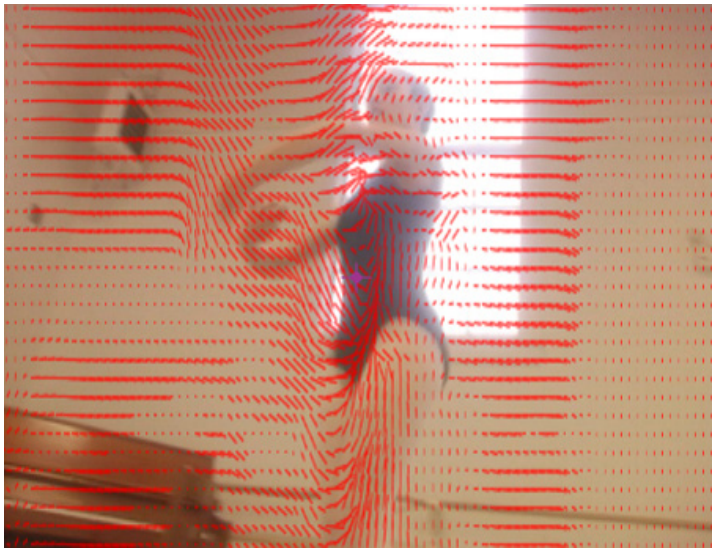
- Depth and 3D structure
  - Two-view stereo
  - Multi-view stereo
  - Shape carving
  - Structure from motion



Thursday + Next Tuesday

What can we get from multiple images?

- Motion
  - Optical flow
  - Tracking
  - Action/activity recognition

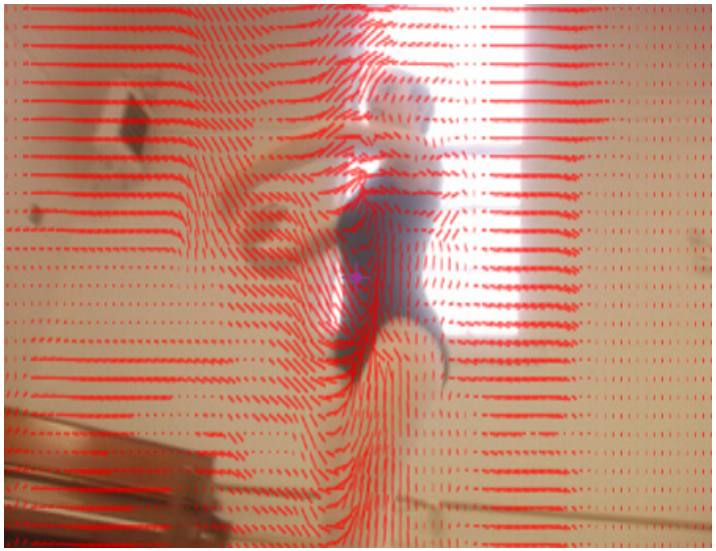


Optical flow (source: http://www.borisfx.com/avid/bccavx/classic_features.php)

Tracking (from Deva Ramanan)

What can we get from multiple images?

- Motion
  - **Optical flow** ← **April 15**
  - Tracking
  - Action/activity recognition



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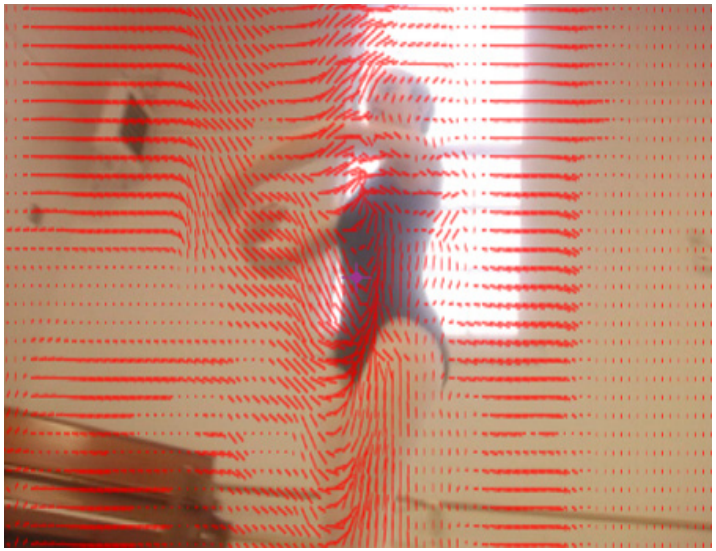
- Motion
  - Optical flow
  - **Tracking** ← **April 20**
  - Action/activity recognition



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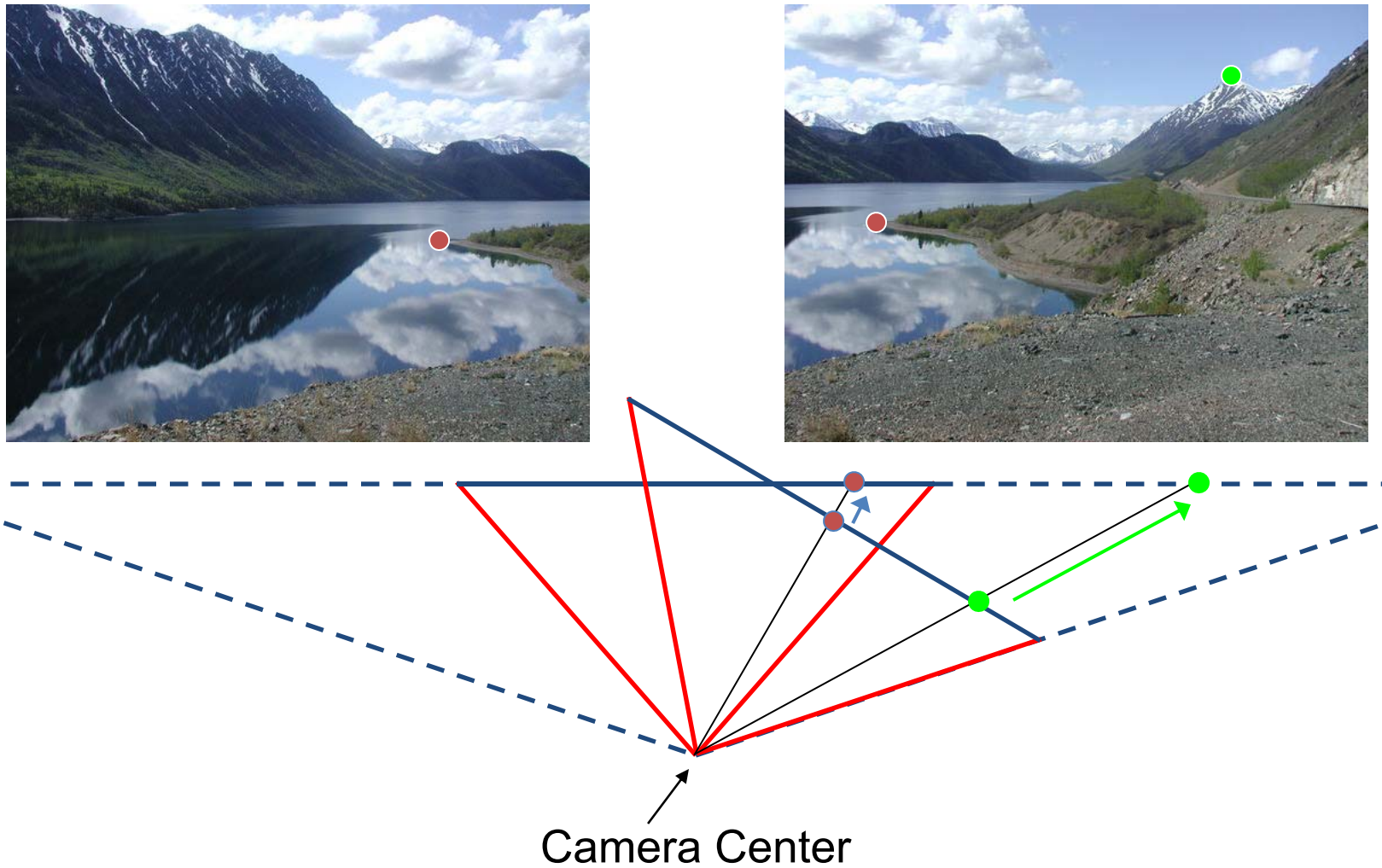
Tracking (from Deva Ramanan)

Today: Image Stitching

- Combine two or more overlapping images to make one larger image

Slide credit: Vaibhav Vaish

Example

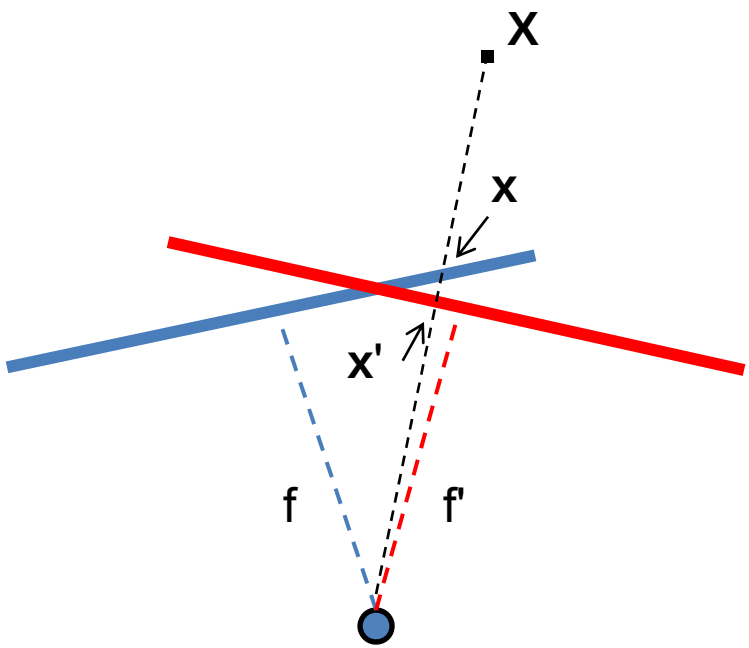


Problem basics

- Do on board

Basic problem

- $x = K\,[R\ t]\,X$
- $x' = K'\,[R'\ t']\,X'$
- $t=t'=0$



- $x'=Hx$ where $H = K'\,R'\,R^{-1}\,K^{-1}$
- Typically only R and f will change (4 parameters), but, in general, H has 8 parameters

Image Stitching Algorithm Overview

1. Detect keypoints
2. Match keypoints
3. Estimate homography with four matched keypoints (using RANSAC)
4. Combine images

Computing homography

- Assume we have four matched points: How do we compute homography **H**?

Direct Linear Transformation (DLT)

$$\mathbf{x}' = \mathbf{H}\mathbf{x} \Rightarrow \mathbf{x}' \times \mathbf{H}\mathbf{x} = \mathbf{0}$$

Only these two provide unique constraints

$$\mathbf{x}' = \begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} \qquad \begin{bmatrix} \mathbf{0}^T & -x'_3\,\mathbf{x}^T & x'_2\,\mathbf{x}^T \\ x'_3\,\mathbf{x}^T & \mathbf{0}^T & -x'_1\,\mathbf{x}^T \\ -x'_2\,\mathbf{x}^T & x'_1\,\mathbf{x}^T & \mathbf{0}^T \end{bmatrix} \mathbf{h} = \mathbf{0}$$

Computing homography

Direct Linear Transform

$$\begin{bmatrix} \mathbf{0}^T & -x'_{13}\,\mathbf{x}_1^T & x'_{12}\,\mathbf{x}_1^T \\ x'_{13}\,\mathbf{x}_1^T & \mathbf{0}^T & -x'_{11}\,\mathbf{x}_1^T \\ \dots & \dots & \dots \\ \mathbf{0}^T & -x'_{n3}\,\mathbf{x}_n^T & x'_{n2}\,\mathbf{x}_n^T \\ x'_{n3}\,\mathbf{x}_n^T & \mathbf{0}^T & -x'_{n1}\,\mathbf{x}_n^T \end{bmatrix} \mathbf{h} = \mathbf{0} \Rightarrow \mathbf{A}\mathbf{h} = \mathbf{0}$$

- Apply SVD: $\boldsymbol{UDV}^T = \boldsymbol{A}$
- $\boldsymbol{h} = \boldsymbol{V}_{\text{smallest}}$ (column of $\boldsymbol{V}$ corr. to smallest singular value)

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_9 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix}$$

Computing homography

- Assume we have four matched points: How do we compute homography **H**?

Normalized DLT

1. Normalize coordinates for each image
   a) Translate for zero mean
   b) Scale so that average distance to origin is sqrt(2)

$$\tilde{\mathbf{x}} = \mathbf{T}\mathbf{x} \qquad \tilde{\mathbf{x}}' = \mathbf{T}'\mathbf{x}'$$

   – This makes problem better behaved numerically (see HZ p. 107-108)
2. Compute **H** using DLT in normalized coordinates
3. Unnormalize: $\mathbf{H} = \mathbf{T}'^{-1}\tilde{\mathbf{H}}\mathbf{T}$

$$\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$$

Computing homography

- Assume we have matched points with outliers: How do we compute homography **H**?

Automatic Homography Estimation with RANSAC

1. Choose number of samples *N*

For probability p of no outliers:

$$N = \log(1-p)/\log(1-(1-\epsilon)^s)$$

- N, number of samples
- s, size of sample set
- ϵ, proportion of outliers

e.g. for $p = 0.95$

Sample size	Proportion of outliers ϵ						
s	5%	10%	20%	25%	30%	40%	50%
2	2	2	3	4	5	7	11
3	2	3	5	6	8	13	23
4	2	3	6	8	11	22	47
5	3	4	8	12	17	38	95
6	3	4	10	16	24	63	191
7	3	5	13	21	35	106	382
8	3	6	17	29	51	177	766

HZ Tutorial '99

Computing homography

- Assume we have matched points with outliers: How do we compute homography **H**?

Automatic Homography Estimation with RANSAC

1. Choose number of samples *N*
2. Choose 4 random potential matches
3. Compute **H** using normalized DLT
4. Project points from **x** to **x'** for each potentially matching pair: $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$
5. Count points with projected distance < t
   - t ~= 6 σ ; σ is measurement error (1-3 pixels)
6. Repeat steps 2-5 *N* times
   - Choose **H** with most inliers

HZ Tutorial '99

Automatic Image Stitching

1. Compute interest points on each image
2. Find candidate matches
3. Estimate homography **H** using matched points and RANSAC with normalized DLT
4. Transform second image and blend the two images
   - Matlab: maketform, imtransform

Some details from a class project

Recognizing Panoramas

Some of following material from Brown and Lowe 2003 talk

Brown and Lowe 2003, 2007

Recognizing Panoramas

Input: N images

1. Extract SIFT points, descriptors from all images
2. Find K-nearest neighbors for each point (K=4)
3. For each image
   a) Select M candidate matching images by counting matched keypoints (m=6)
   b) Solve homography $\mathbf{H}_{ij}$ for each matched image

Recognizing Panoramas

Input: N images

1. Extract SIFT points, descriptors from all images
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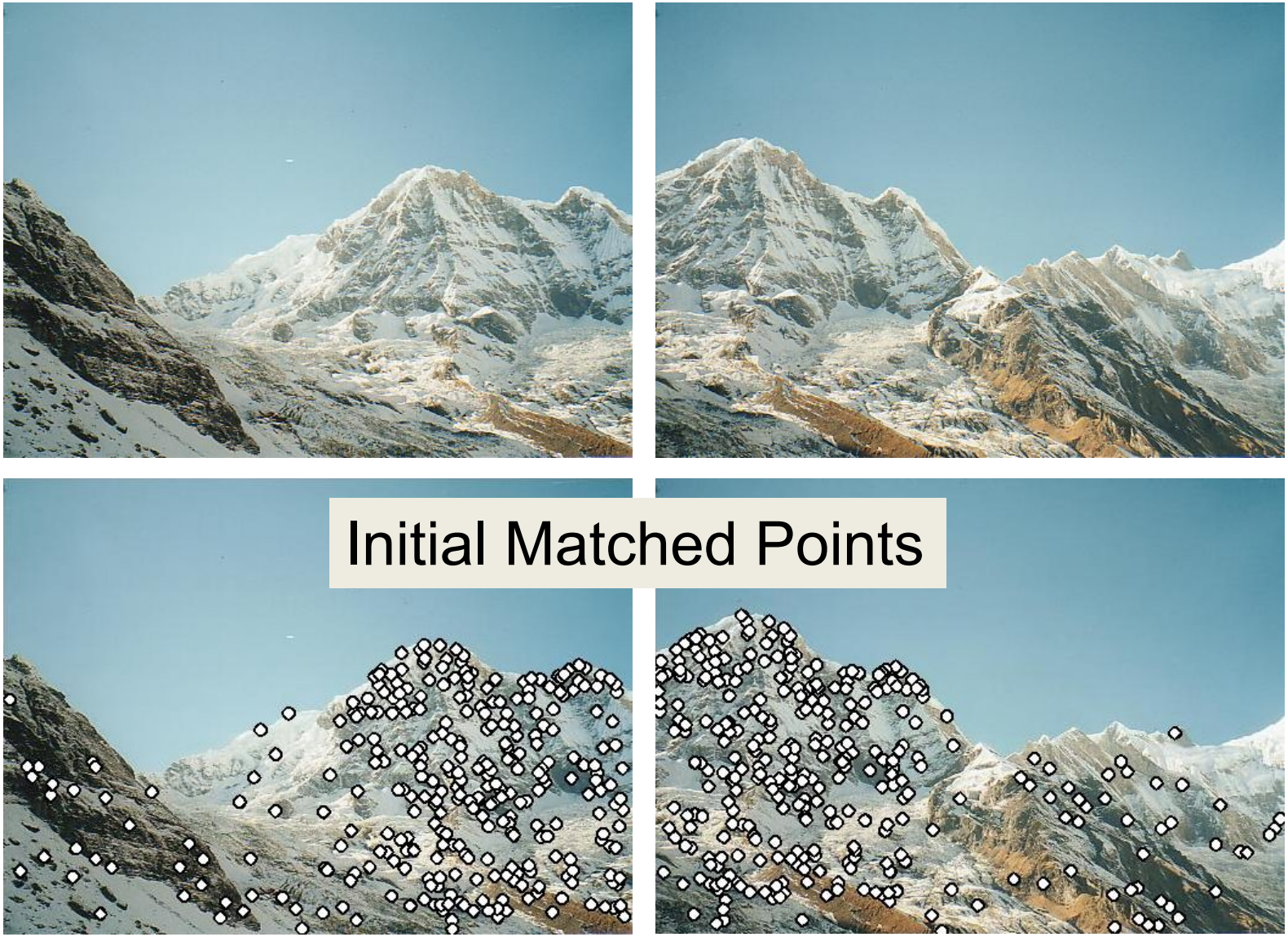
   a) Select M candidate matching images by counting matched keypoints (m=6)
   b) Solve homography $\mathbf{H}_{ij}$ for each matched image
   c) Decide if match is valid ($n_i > 8 + 0.3\ n_f$)

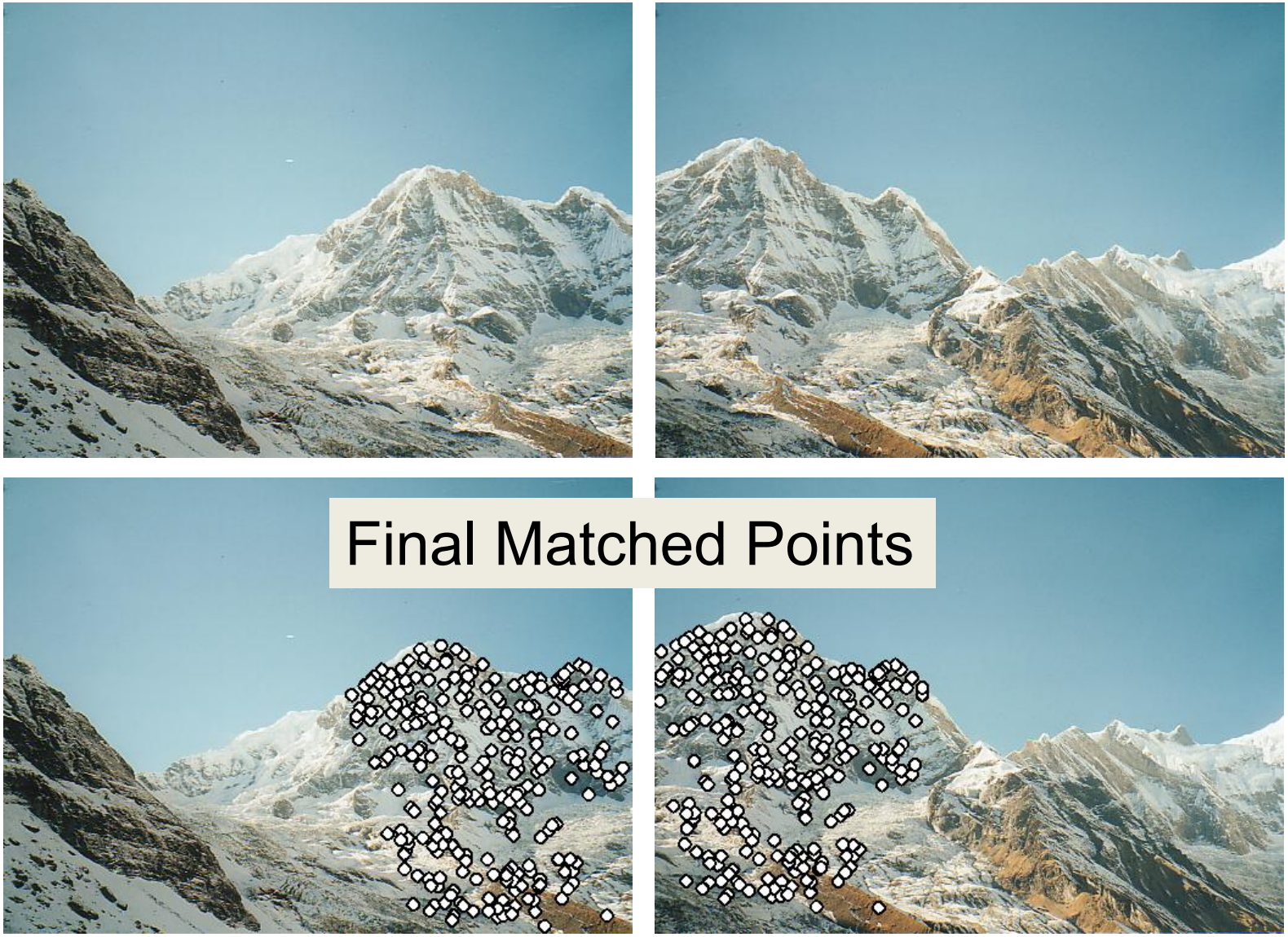
inliers

features in overlapping area

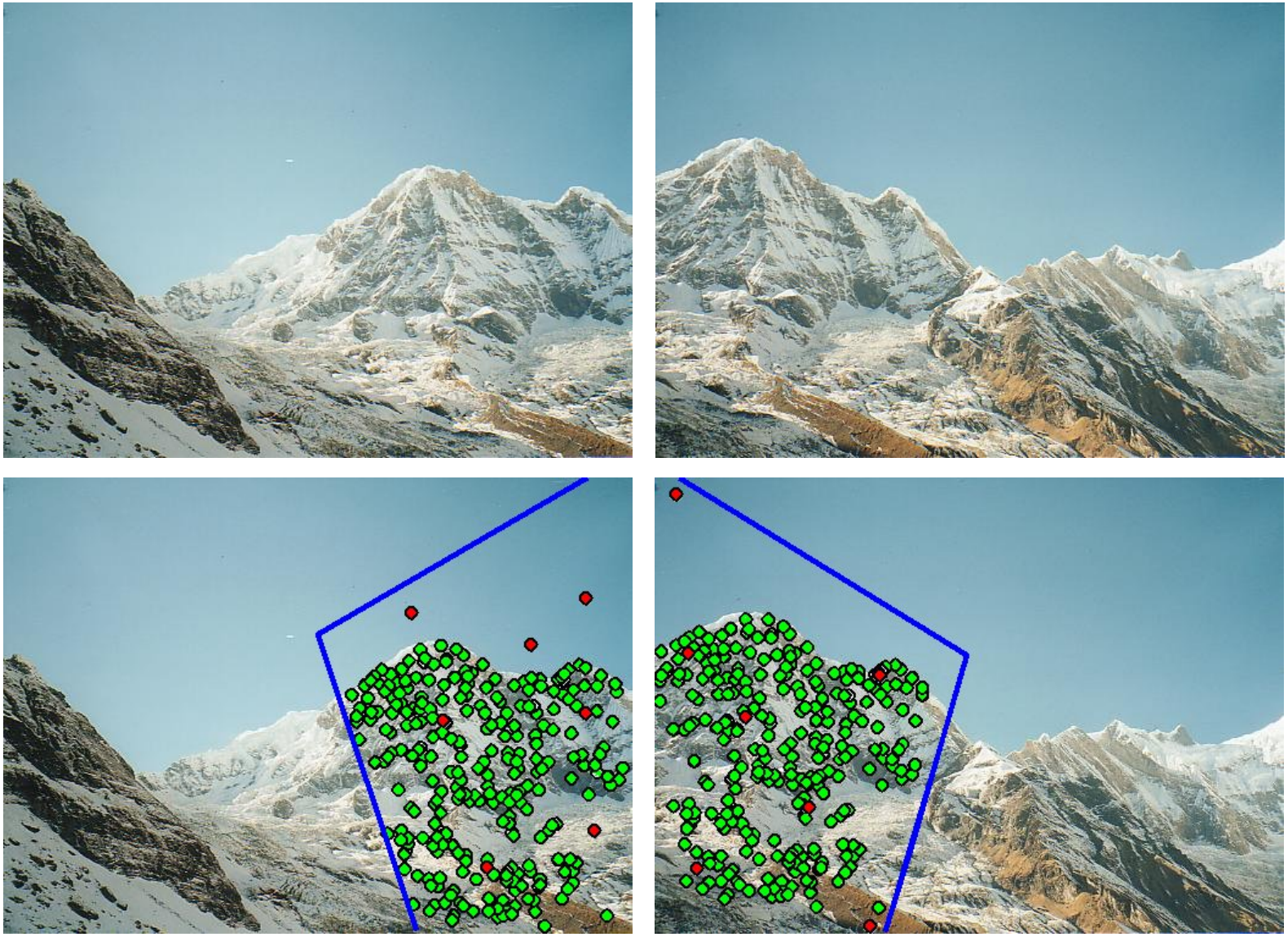
RANSAC for Homography



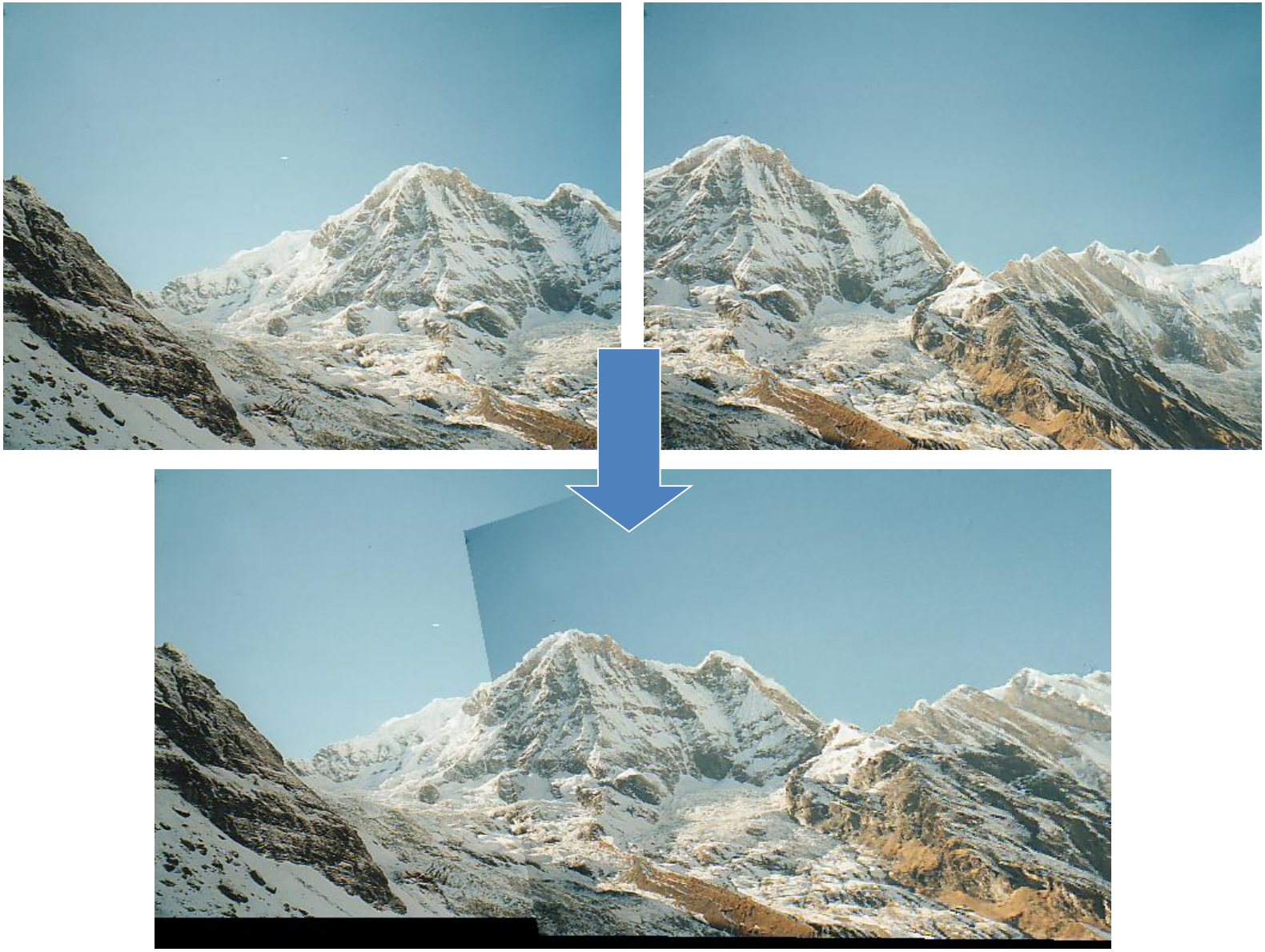
RANSAC for Homography



Verification



RANSAC for Homography

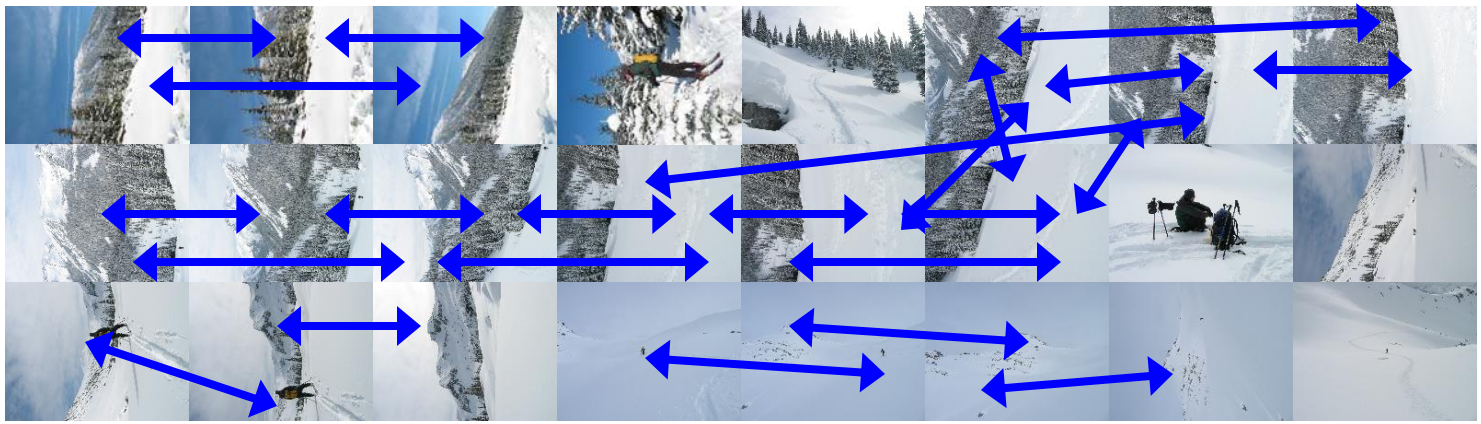


Recognizing Panoramas (cont.)

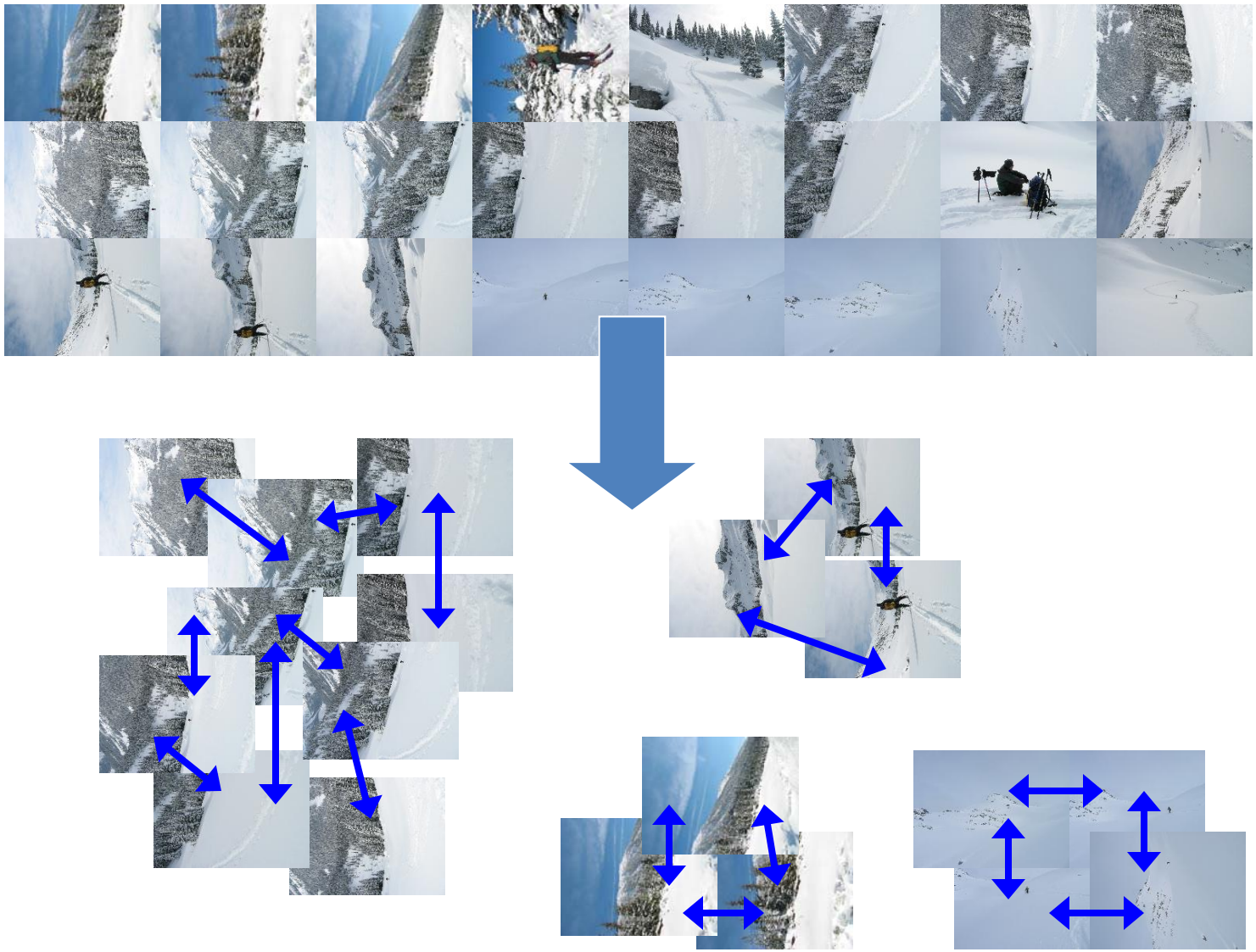
(now we have matched pairs of images)

4. Find connected components

Finding the panoramas



Finding the panoramas



Recognizing Panoramas (cont.)

(now we have matched pairs of images)

4. Find connected components
5. For each connected component
   a) Perform bundle adjustment to solve for rotation (θ_1, θ_2, θ_3) and focal length f of all cameras
   b) Project to a surface (plane, cylinder, or sphere)
   c) Render with multiband blending

Bundle adjustment for stitching

- Non-linear minimization of re-projection error

$$\mathbf{R}_i = e^{[\boldsymbol{\theta}_i]_\times}, \ [\boldsymbol{\theta}_i]_\times = \begin{bmatrix} 0 & -\theta_{i3} & \theta_{i2} \\ \theta_{i3} & 0 & -\theta_{i1} \\ -\theta_{i2} & \theta_{i1} & 0 \end{bmatrix}$$

- $\hat{\mathbf{x}}' = \mathbf{H}\mathbf{x}$ where **H = K' R' R^{-1} K^{-1}**

$$\mathbf{K}_i = \begin{bmatrix} f_i & 0 & 0 \\ 0 & f_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$error = \sum_1^N \sum_j^{M_i} \sum_k dist(\mathbf{x}', \hat{\mathbf{x}}')$$

- Solve non-linear least squares (Levenberg-Marquardt algorithm)
  - See paper for details

Bundle Adjustment

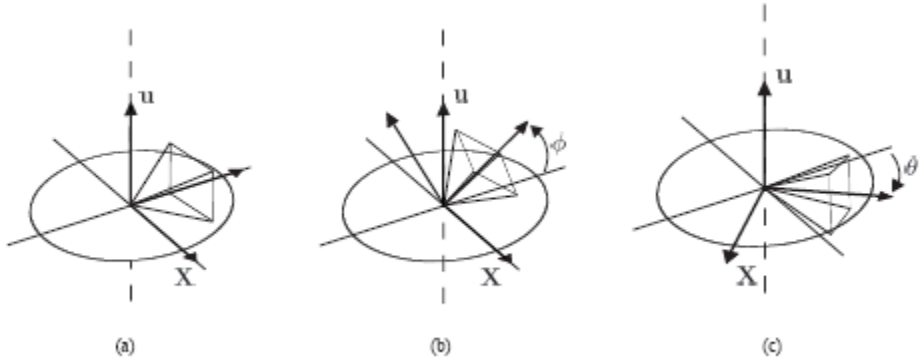
- New images initialised with rotation, focal length of best matching image

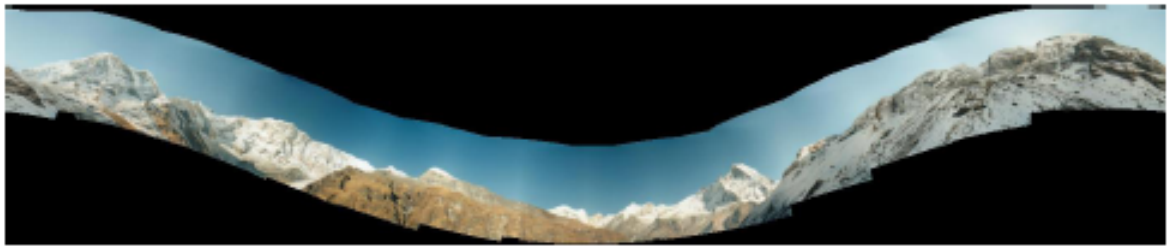
Bundle Adjustment

- New images initialised with rotation, focal length of best matching image

Straightening

- Rectify images so that “up” is vertical





(a) Without automatic straightening

(b) With automatic straightening

Blending

- Gain compensation: minimize intensity difference of overlapping pixels

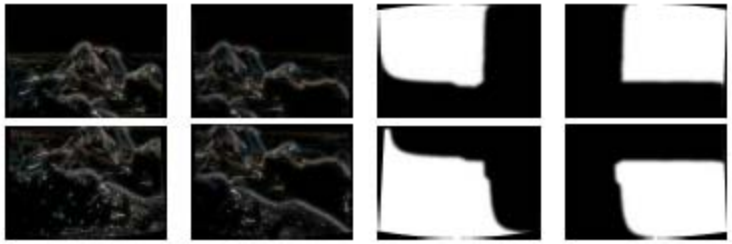
- Blending
  - Pixels near center of image get more weight
  - Multiband blending to prevent blurring

Multi-band Blending

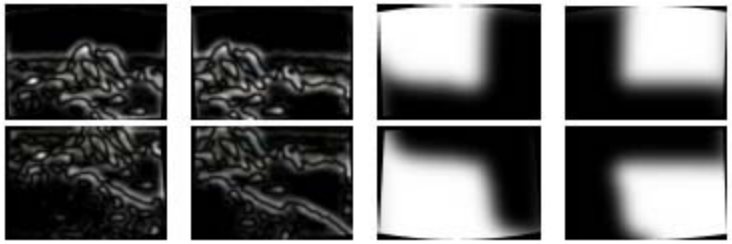
- Burt & Adelson 1983
  - Blend frequency bands over range $\propto \lambda$

Multiband blending

(a) Original images and blended result



(b) Band 1 (scale 0 to σ)



(c) Band 2 (scale σ to 2σ)

(d) Band 3 (scale lower than 2σ)

Blending comparison (IJCV 2007)

(a) Linear blending

(b) Multi-band blending

Blending Comparison

(b) Without gain compensation

(c) With gain compensation

(d) With gain compensation and multi-band blending

Further reading

- DLT algorithm: HZ p. 91 (alg 4.2), p. 585
- Normalization: HZ p. 107-109 (alg 4.2)
- RANSAC: HZ Sec 4.7, p. 123, alg 4.6

- Recognising Panoramas: Brown and Lowe, IJCV 2007 (also bundle adjustment)

Things to remember

- Homography relates rotating cameras
- Recover homography using RANSAC and normalized DLT
- Bundle adjustment minimizes reprojection error for set of related images
- Details to make it look nice (straightening, blending)